RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR

B.A./B.SC. FIRST SEMESTER (July – December) 2014

Mid-Semester Examination, September 2014

Date : 18/09/2014

MATHEMATICS (General)

Time : 12 noon – 1 pm

Paper : I

Full Marks : 25

Group – A

Answer **any two** questions :

1. Show that the product of all the values of $(1+i\sqrt{3})^{3/4}$. [4]
2. State De Moivre's theorem. Find the principal value of $(1+i)^i$. [1+3]
3. State the Rolle's theorem. Use it to find the range of values of K for which the equation $x^4 - 14x^2 + 24x - K = 0$ has four real unequal roots. [1+3]

Group - B

Answer **any two** questions :

4. State De Morgan's Law. Verify it for the sets A & B, where A = {1,5,6}, B = {5,6,7} and U = {1,2,3,4,5,6,7} as universal set. [1+3]
5. a) If $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x) = x^2$ and $g:\mathbb{R}\to\mathbb{R}$ by $g(x) = e^x$, find $g \circ f$ and $f \circ g$. [1]

   b) If $f:A\to B$ and $g:B\to C$ be both injective mappings then prove that $g\circ f : A \to C$ is injective. [3]
6. Prove that in a group $(G,*)$, $(a*b)^{-1} = b^{-1}*a^{-1}\ \forall a,b\in G$. Define subgroup of a group. [3+1]

Group – C

7. Answer **any one** question :

   a) If $u = \sin^{-1}\sqrt{\dfrac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$ then, using Euler's theorem prove that :

   i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = -\dfrac{1}{12}\tan u$

   ii) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = \dfrac{\tan u}{12}\left[\dfrac{13}{12} + \dfrac{\tan^2 u}{12}\right]$ [2+3]

   b) State Schwarz theorem.

   If $f(x,y) = xy\left(\dfrac{x^2-y^2}{x^2+y^2}\right), x\neq 0, y\neq 0$

   $= 0, \qquad x = 0, y = 0$

   Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$. [1+4]

8. Answer **any one** question :

   a) If $f(x,y) = x\sin\dfrac{1}{y} + y\sin\dfrac{1}{x}, xy\neq 0$

   $= 0 \qquad , xy = 0$

   prove that the repeated limits do not exist at the origin. Also, show analytically that the double limit of f(x,y) exists at the origin. [1+3]

   b) If $\phi(cx-az, cy-bz) = 0$, using Chain Rule, show that $ap+bq=c$, where $p=\dfrac{\partial z}{\partial x}$, $q = \dfrac{\partial z}{\partial y}$. [4]

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